

Steady-state signatures of radiation trapping by cold multilevel atoms

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In this paper, we use steady-state measurements to obtain evidence of radiation trapping in an optically thick a cloud of cold rubidium atoms. We investigate the fluorescence properties of our sample, pumped on opened transitions. The intensity of fluorescence exhibits a non trivial dependence on the optical thickness of the media. A simplified model, based on rate equations self-consistently coupled to a diffusive model of light transport, is used to explain the experimental observations in terms of incoherent radiation trapping on one spectral line. Measurements of atomic populations and fluorescence spectrum qualitatively agree with this interpretation.

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I. INTRODUCTION

Trapping of light due to multiple scattering by the atoms of a gas plays an important role in many transfer problems ranging from plasma physics to astrophysics [1]. Moreover, atomic vapors provide a well characterized set of identical and very efficient resonant scatterers; from the beginning of the 20th century, they have been extensively used to experimentally investigate the phenomenon of radiation trapping [2–4]. In hot atomic vapours, frequency redistribution due to atomic motion significantly affects the radiation trapping process [5] leading to non diffusive multiple scattering processes [6]. From that prospect, samples of cold atoms, where frequency redistribution is strongly reduced, provide a interesting model experiment to study the role of multiple scattering with increasing complexity. Time resolved experiments on a closed atomic transition have demonstrated that cold atoms give raise to very efficient radiation trapping [7–9], consistent with steady state experiments measuring diffuse reflection or transmission measurements [10]. Even more subtle interference effects have been observed such as coherent backscattering [11] opening the way to study effects such localization of light in cold atomic vapours [12].

Another additional interesting feature is to provide light amplification when atomic vapours are optically pumped under certain conditions [13–18]. For a given level of gain, when feedback due to multiple scattering is increased, a runaway regime can be reached where gain in the volume compensates for losses through the surface: then, the power emitted by the sample increases until saturation of gain is reached. This is the photonic bomb predicted by Letokhov[19]. Above threshold, modal selection can occur, feedback due to multiple scattering favourising certain frequencies or directions of emission[20]. It has been predicted that gain and feedback due to multiple scattering can be combined to obtain random lasing action in a cloud of cold atoms[21].

In the context of quantum information, radiation trap-

ping has been studied as a perturbative phenomena limiting the atomic coherence [22]. It has also been shown that radiation trapping inside the media can impact the photon statistics of light emitted outside the sample[22, 23]. Finally, mechanical effects of trapped light also have been investigated and limits the density of atoms [24] or even induce mechanical instabilities [25].

In this paper, we consider a configuration identified as a good candidate to obtain random lasing action, where multilevel atoms of rubidium are optically pumped on an open transition to give rise to Raman gain. We show that even in the absence of gain, light trapped inside the media can have a significant influence on the atomic populations, and therefore on the intensity and spectra of the emission of the sample. Understanding this regime is a required preliminary condition to later identify signatures of a random laser in this configuration. We stress that we exhibit here a model experiment allowing to investigate in laboratory environment features of radiation transfer in a multi-line system out of thermal equilibrium - well known in the domain of astrophysics [26–28].

The paper is organized as follows: section II describes the experimental setup, the atomic configuration under consideration and the main measured signatures of radiation trapping. In section III, we introduce a qualitative model (section III.A) based on a diffusion equation for light transport and rate equations for the atomic response to assess the influence of trapped light on the atomic emission. Detailed equations and the self-consistent solution of the coupled equations of the atomic response and a diffusion equation for light are presented in section III.B, together with comparison the experimental results. Some approximations done in this simplified model are discussed in section III.C, before we conclude in section IV.

II. EXPERIMENT

The main features of our experiment have already been described elsewhere [29]. We use 6 counter-propagating trapping beams with a waist of 3.4cm to load atoms of ^{85}Rb from a vapor in a magneto-optical trap (see figure 1). Trapping beams are detuned by -3Γ from the $F = 3 \rightarrow F' = 4$ hyperfine transition of ^{85}Rb , where Γ is the width of the transition. Six additional repumper beams tuned to the $F = 2 \rightarrow F' = 3$ transition maintain most of the atomic populations in $F = 3$. The number of atoms loaded can be varied between roughly 10^8 and 10^{11} by changing the background vapor pressure ($\sim 10^{-8}$ to a few 10^{-7} mbar) and on the time duration chosen to load the trap (~ 10 to 500 ms). Once the atoms are loaded, we perform a temporal dark MOT by reducing the intensity of the repumper beams. This leads to a reduction of the size of the cloud and an increased spatial and optical density of the cloud. By varying the duration of this compression process and with 10^9 atoms initially loaded, it is possible to adjust the optical thickness b_0 of the sample ($b_0 = -\ln(T)$ where T is the coherent forward transmission measured on the $3 \rightarrow 4'$ resonance) in a range of 20 – 75. We stress that this protocol allows for changing the optical thickness while keeping the number of atoms quasi-constant ($\pm 10\%$). After this dark MOT period, the trapping lasers and magnetic field gradients are switched off and two contra-propagating pumps P with a waist of 2.4cm and a center intensity of 1.9 mW/cm^2 each are shone on the sample. The pump beam is obtained from a master laser which is then amplified by two stages of saturated slave lasers; hence, by tuning the frequency of the master laser using a double-pass acousto-optics modulator, we can scan the detuning δ_P of the pump with respect to the $F = 3 \rightarrow F' = 2$ transition by more than 16Γ with altering its intensity by less than 0.1%. Note that when the pump is detuned from the $F = 3 \rightarrow F' = 2$ resonance, Raman gain ($F = 3 \rightarrow F' = 2 \rightarrow F = 2$) can be obtained in this system [15, 16, 30–32] when the population of the $F = 3$ state is larger than that of the $F = 2$ state. Using a repumper on the $F = 2 \rightarrow F' = 3$ transition to control the population balance between the hyperfine ground states of ^{85}Rb a steady state regime can be reached. In the work described in this paper, we use a weak repumper, so that most atoms are in the $F = 2$ ground state. Finally, fluorescent emission from our sample is gathered in a solid angle of 0.01sr and measured using a high gain photodiode. Our measurements are typically made on a time scale of a few $100\mu\text{s}$ (typically 1 or 2 ms), while the system typically reaches its steady-state in roughly $1/\Gamma = 26\text{ns}$ for one atom physics and approximatively few μs for radiation trapping with our experimentally optical thickness [9]. Hence, all observations reported here correspond to a steady-state regime.

The most striking result of this experiment is shown on figure 2. We measure the fluorescence intensity of our sample submitted to a pump and to a weak and detuned repumper, as a function of the detuning of the pump. We

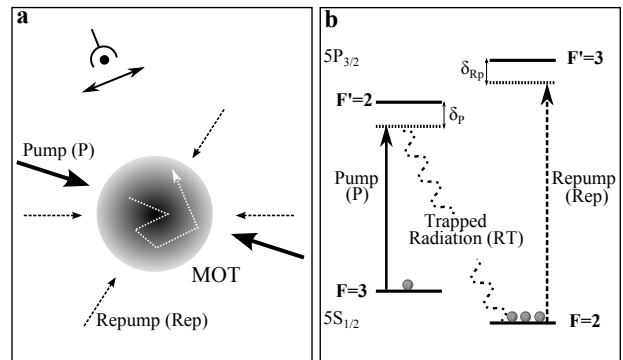


FIG. 1: Experimental setup. A cloud of ^{85}Rb is formed in a magneto-optical trap, whose optical thickness can be adjusted by varying dark MOT parameters, while keeping constant the number of atoms. A pump (P) ($3 \rightarrow 2'$ transition) and a repumper (Rep) ($2 \rightarrow 3'$ transition) are then used to illuminate the atoms, while fluorescent emission from the sample is gathered by a lens in a large solid angle and its intensity measured using a high gain photodiode.

have paid particular care to keep the number of atoms constant ($\pm 10\%$) as we change the optical thickness of the cloud. This protocol allows to easily distinguish collective effects from a change of fluorescence due to an increased number of atoms. With this protocol, if atoms would react individually to the exciting beams, the detected fluorescence would remain constant as we change the optical thickness b_0 . As one can clearly see in figure 2, we observe a strong increase in this intensity with b_0 when the pump is tuned to the $F = 3 \rightarrow F' = 2$ resonance. This is a signature of a strong collective emission effect that occurs in our sample.

III. SELF-CONSISTENT MODEL OF ATOMIC RESPONSE COUPLED TO RADIATION TRANSFER OF LIGHT

A. Qualitative explanation

In what follows, we will show this result can be understood by taking into account the strong impact of diffuse light on the atomic populations. A qualitative description allows to understand the dominant phenomenon of this situation and involves a description of the atomic response and a diffusion equation for a particular atomic emission line.

The atomic response can be obtained using optical Bloch equations, describing atomic populations and (optical and Zeeman) coherences. For a two level transition excited with a single optical frequency, steady state results for populations and emission intensities can be obtained from rate equations, allowing a simpler set of equations to be used. Note that a similar simplification can be used for time dependant quantities if the decay

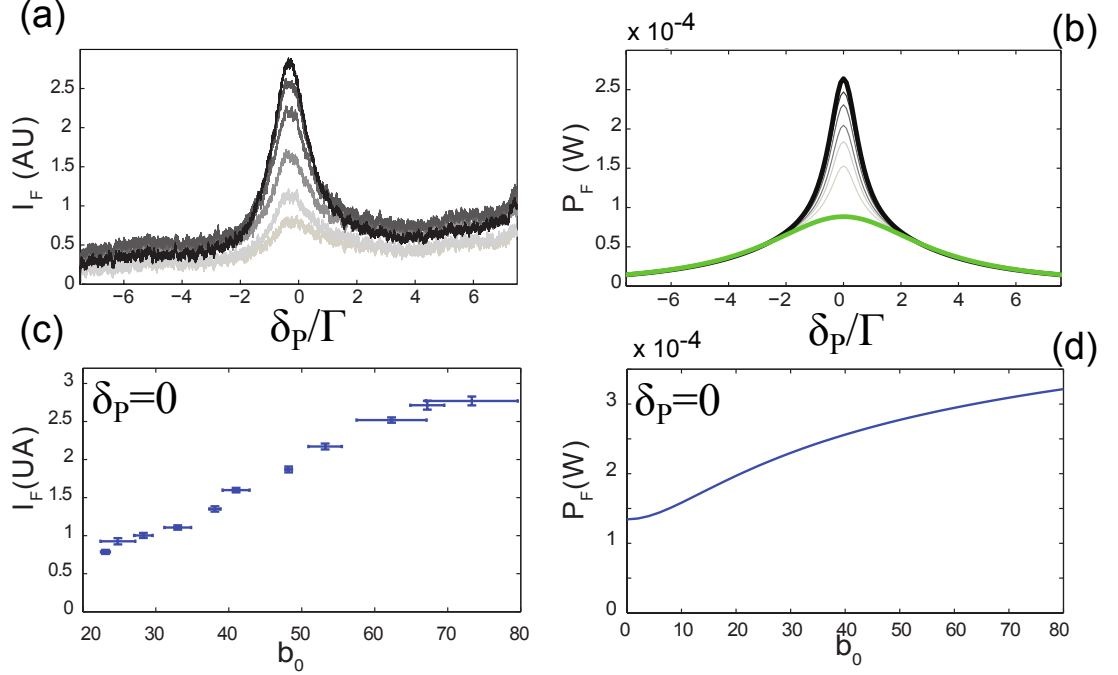


FIG. 2: Evolution of the intensity of fluorescence with the optical thickness, for a sample of fixed (1.2×10^9) number of atoms. **a.** Intensity of fluorescence, measured experimentally, as a function of pump detuning, for various optical thicknesses b_0 : from light to dark grey, 23, 33, 41, 53, 62 and 73. A clear increase of fluorescence for zero detuning of the pump is observed, that is due only to a collective emission effect. Pump intensity: 1.88 mW/cm^2 , repumper intensity: 0.48 mW/cm^2 , repumper detuning from $2 - 3'$ transition: 4Γ . **b.** Total power emitted by the sample in the same conditions, estimated from our model coupling diffusion and rate equations. **c.** Evolution with the optical thickness b_0 of the intensity of fluorescence measured experimentally for resonant pump. **d.** Total power emitted by the sample, estimated from our model, in good qualitative agreement with experimental observations.

rate of atomic coherences is much larger than those of populations, a situation which can e.g. occur in presence of collisions. The stationary solution of the rate equations is given by $\rho_{ee} = \frac{1}{2} \frac{s}{1+s}$, where $s = 2BI/\Gamma$ is the detuning dependent saturation parameter, with the Einstein coefficient B (see details below) and the incident laser intensity I .

If radiation trapping effects can be neglected, our atomic four level scheme (figure 1) reduces to two 2-level atoms excited each by a laser, allowing to use rate equations for the steady state populations. These two 2-level systems are coupled by incoherent radiative decay (from $F' = 2 \rightarrow F = 2$ and $F' = 3 \rightarrow F = 3$).

In the experiments described in this paper we focus on a situation with a strong pump laser tuned close to the atomic line $F = 3 \rightarrow F' = 2$ and a weaker repumper laser detuned from the atomic line $F = 2 \rightarrow F' = 3$. In this case most atoms are in the ground state $F = 2$. We measure the total fluorescence P_F of the atomic cloud, which is proportional to the total excited state

population $\rho_{2'2'} + \rho_{3'3'}$ of states $F' = 2$ and $F' = 3$. If the two transitions are treated independantly, we get for P_F :

$$P_F \propto \rho_{2'2'} + \rho_{3'3'} = \rho_{33} \frac{1}{2} \frac{s_P}{1 + s_P} + \rho_{22} \frac{1}{2} \frac{s_{Rep}}{1 + s_{Rep}}, \quad (1)$$

where $s_P = \frac{I_P/I_{sat,P}}{1+4(\delta_P/\Gamma)^2}$ and $s_{Rep} = \frac{I_{Rep}/I_{sat,Rep}}{1+4(\delta_{Rep}/\Gamma)^2}$ are the saturation parameters of the two transitions (with intensities I_P, I_{Rep} and detunings δ_P, δ_{Rep}), $I_{sat,P}$ and $I_{sat,Rep}$ are the saturation intensity of the transitions and ρ_{22}, ρ_{33} the populations of the ground states. When the transition $F = 3 \rightarrow F' = 2$ is saturated ($s_P \gg 1$) and the transition $F = 2 \rightarrow F' = 3$ is weakly driven ($s_{Rep} \ll 1$), this expression can be approximated by:

$$P_F \propto \frac{\rho_{33}}{2} + \frac{\rho_{22}}{2} s_{Rep} \quad (2)$$

As we have $\rho_{33} + \rho_{22} \approx 1$ and $s_{Rep} \ll 1$, a transfer of

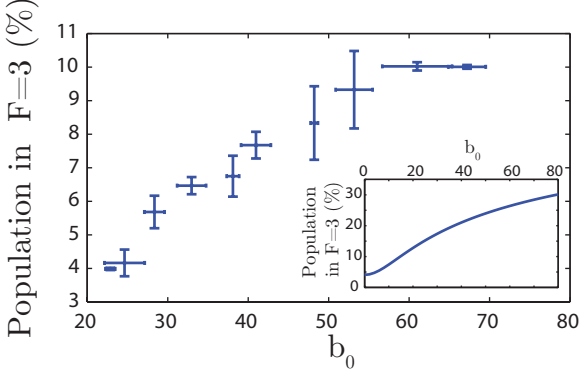


FIG. 3: Atomic population in the $F=3$ state, as a function of the optical thickness, estimated by absorption imaging. Pumping conditions are the same as for figure 2, and the pump is on resonance. A clear increase of the population of $F = 3$ with the optical thickness is demonstrated. **Inset** : Evolution with b_0 of the population of the $F = 3$ state predicted by our model. The qualitative trend is reproduced, but this population is overestimated for large optical thicknesses.

populations from the $F = 2$ state to the $F = 3$ state will increase the total emission.

When a resonant pump is sent on the open $F = 3 \rightarrow F = 2'$ transition, atoms are efficiently depumped into the $F = 2$ state. An atom can generate fluorescence only when, occasionally, a photon from the weak and detuned repumper is scattered. If the scattering is inelastic, the $F=3$ state might be temporarily populated, and fluorescence will be generated by scattering of the pump on the $F = 3 \rightarrow F' = 2$ line. Yet the atom will quickly go back to the $F = 2$ state, emitting one photon on the $F' = 2 \rightarrow F = 2$ line. In a cloud of low optical thickness, this photon is likely to exit the sample and the process generating fluorescence ends there. However, in a cloud of high optical thickness, light emitted on the $F' = 2 \rightarrow F = 2$ line is likely to excite a neighbouring atom initially in the $F = 2$ state. It acts as an additional repumper that increases the population in the $F = 3$ state and therefore the amount of atoms available for the very efficient scattering process of the pump. As a consequence, the total intensity of fluorescence exiting the sample increases with optical thickness, dominated by an emission on the $F = 3 \rightarrow F' = 2$ line.

Additional measurements confirm this scenario. Using standard absorption imaging, we can measure the fraction of atoms in the $F = 3$ state once the pump and repumper have been switched off. As shown on figure 3, a clear increase of the population of the $F = 3$ state with the optical thickness is observed. Another signa-

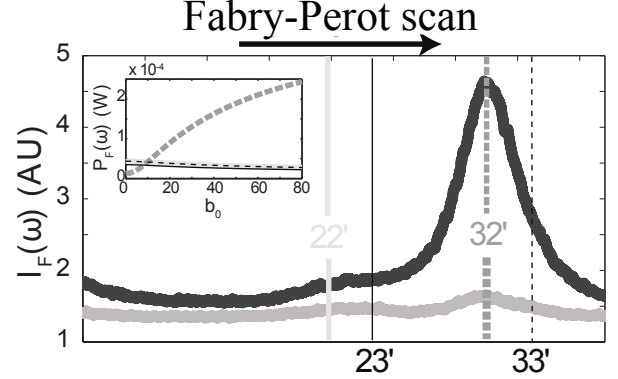


FIG. 4: Spectrum of fluorescence obtained by scanning the length of a Fabry-Perot cavity, for an on resonance optical thickness of 50 (light grey) and 145 (dark grey). Pumping conditions are identical to figure 3, but the number of atoms is now $1.4 \cdot 10^{10}$ - this to maximize signal at the output of the cavity. A clear increase with the optical thickness of the intensity emitted on the $F = 3 \rightarrow F' = 2$ line is observed. **Inset**: Evolution of the intensity emitted on each hyperfine line of ^{85}Rb D2 line as a function of the optical thickness, estimated with our model. A slight decrease of all lines but the $F = 3 \rightarrow F' = 2$ (thick dotted grey) is predicted, which is compatible with experimental observations.

ture is obtained by measuring the spectra of the light exiting the sample. Using a Fabry-Perot cavity with a free spectral range of 400MHz and a very low finesse of 14 we have measured the spectrum of the emitted light by coupling the quasi-isotropic emission from the sample to the cavity. With a resolution of $\approx 30\text{MHz}$ the different hyperfine lines of ^{85}Rb can be resolved. As one can see in figure 4 we observe a strong increase of the emission on the $F = 3 \rightarrow F' = 2$ state for larger optical thickness of the sample. Both these results confirm the qualitative explanation of the increase of fluorescence intensity with b_0 explained by the enhanced repumping of atoms into the $F = 3$ ground state with larger excitation rates.

B. Detailed description of the model

To go beyond this qualitative description and to assess to what extent the population redistribution due to the diffuse light in the sample can explain the observed effects, we introduce in this section an ab initio model of radiation trapping in the steady state regime.

In what follows, we assume that only light with a frequency close to the $F = 2 \rightarrow F' = 2$ transition can be trapped in the system and influence the atomic popu-

lation balance. This assumption is reasonable at least for intermediate optical thickness, since repumper light scattered on the $F = 3 \rightarrow F' = 3$ and $F = 2 \rightarrow F' = 3$ is detuned by several line widths Γ from any resonance ($\delta_{Rep} = -4\Gamma$). Furthermore light scattered on the $F = 3 \rightarrow F' = 2$ transition is subject to low optical thickness, as the $F = 3$ state is weakly populated (at least before population redistribution occurs due to radiation trapping). We model the light scattered by the atoms at a frequency close to the $F = 2 \rightarrow F' = 2$ resonance and trapped inside the media by an incoherent pumping rate. The validity of this assumption will be further discussed at the end of this paper. Finally, we only consider 4 effective nondegenerate hyperfine levels and do not take into account the complex Zeeman substructure of the different hyperfine levels. We thus neglect polarisation effects of the scattered light [28] or the possibility of dark states due to a coherence between different Zeeman sublevels.

Since only two independent and non degenerate transitions are coherently excited (the trapped light is modeled by a rate equation), stationary optical Bloch equations reduce to stationary rate equations:

$$\begin{aligned}\dot{\rho}_{22} &= 0 = -B_{23'}(\delta_{Rep})I_{Rep}(\rho_{22} - \rho_{3'3'}) \\ &\quad - B_{22'}(\delta_{RT})I_{RT}(\rho_{22} - \rho_{2'2'}) \\ &\quad + \Gamma(T_{3'2}\rho_{3'3'} + T_{2'2}\rho_{2'2'}) \\ \dot{\rho}_{33} &= 0 = -B_{32'}(\delta_P)I_P(\rho_{33} - \rho_{2'2'}) \\ &\quad + \Gamma(T_{3'3}\rho_{3'3'} + T_{2'3}\rho_{2'2'}) \\ \dot{\rho}_{2'2'} &= 0 = B_{32'}(\delta_P)I_P(\rho_{33} - \rho_{2'2'}) \\ &\quad + B_{22'}(\delta_{RT})I_{RT}(\rho_{22} - \rho_{2'2'}) - \Gamma\rho_{2'2'} \\ \dot{\rho}_{3'3'} &= 0 = B_{23'}(\delta_{Rep})I_{Rep}(\rho_{22} - \rho_{3'3'}) - \Gamma\rho_{3'3'} \\ 1 &= \rho_{2'2'} + \rho_{3'3'} + \rho_{22} + \rho_{33}\end{aligned}\quad (3)$$

where ρ_{ii} is the population of state i and $T_{i'j}$ the branching ratio for desexcitation from state i' to state j accounting for the degeneracy of states i' and j when assuming a statistical population in all Zeeman sublevels. I_P , I_{Rep} , I_{RT} are respectively the intensities of the pump, repumper, and diffuse light, and δ_P , δ_{Rep} , δ_{RT} their respective detuning to the transitions they excite. Finally, the detuning dependant Einstein coefficients B_{ij} writes :

$$B_{ij}(\delta) = \frac{1}{3} \frac{2F' + 1}{2F + 1} T_{ij} \frac{\sigma_0}{\hbar\omega_0} \frac{1}{1 + 4\frac{\delta^2}{\Gamma^2}} \quad (4)$$

where $\sigma_0 = 3\lambda_0^2/2\pi$ is the on resonance cross section of a two level atom with a transition at frequency $\omega_0 = 2\pi c/\lambda_0$. For our level scheme, the values of $T_{ij} = \frac{2J'+1}{2J+1} \frac{2F'+1}{2F'+1} S_{F'F}$ are : $T_{2'2} = 1 - T_{2'3} = 14/18$ and $T_{3'3} = 1 - T_{3'2} = 10/18$, where $S_{F'F}$ are the strength factors of the transitions[33].

As pump and repumper beams are weakly scattered, the corresponding intensities I_P and I_{Rep} can be considered homogeneous. On the other hand, the intensity I_{RT} of diffuse light at $\omega \approx \omega_{22'}$ is not known and can strongly

vary in space. In the stationary regime, we compute its profile by solving a diffusion equation:

$$\frac{1}{3n\sigma_{ext}(\mathbf{r})} \Delta I_{RT}(\mathbf{r}) = -nW_{RT}(\mathbf{r}), \quad (5)$$

where \mathbf{r} is the position in the cloud, and $I_{RT}(\mathbf{r})$, $W_{RT}(\mathbf{r})$ and $\sigma_{ext}(\mathbf{r})$ are the spatially dependant intensity of the diffuse light, the electromagnetic power around the $F = 2 \rightarrow F' = 2$ frequency and the extinction cross section, which are obtained from the self-consistent solution of eqs. 3 and 5. We have assumed the spatial density of atoms n to have a homogeneous spherical symmetry. The solution of the self-consistent model thus depends on the optical thickness b_0 via: $b_0 = \sigma_0 n^{2/3} N^{1/3} (6/\pi)^{1/3}$, where N is the total number of atoms, which is kept constant. We note that the prefactor $1/(3n\sigma_{ext}(\mathbf{r}))$ amounts to a diffusion equation evaluated in absence of absorption [34, 35].

We can now express σ_{ext} and W_{RT} as a function of the atomic populations by:

$$\begin{aligned}\sigma_{ext}(\mathbf{r}) &= \frac{2F' + 1}{2F + 1} \frac{T_{2'2}\sigma_0}{1 + 4(\delta_P/\Gamma)^2} (\rho_{2'2'}(\mathbf{r}) - \rho_{22}(\mathbf{r})) \\ W_{RT}(\mathbf{r}) &= \hbar\omega_0 \Gamma T_{22'} \rho_{22'}(\mathbf{r}) - \sigma_{ext}(\mathbf{r}) I_{RT}(\mathbf{r})\end{aligned}\quad (6)$$

Note that the source term $W_{RT}(\mathbf{r})$ in eq. 5 includes the emitted power on the $F = 2' \rightarrow F = 2$ line and the attenuation (extinction) of the incident intensity at this frequency. In contrast to the situation where the emitted light can be understood as the attenuation of an external laser, we do not have an incident laser on the $F = 2' \rightarrow F = 2$ line. The energy of the diffuse light is taken from the light scattered by other atoms requiring to add this extinction term for $W_{RT}(\mathbf{r})$. In the case of a two level atoms without pump, emitted power for instance exactly balances the extinction and a diffusion equation without source term is the adequate description.

To simplify the algebra, we consider a spherical cloud, so that the problem becomes rotation invariant, and the diffusion equation reduces to a one dimensional equation in spherical coordinates. The coupled eqs. 3 and 5 are solved in a self-consistent way, using an iterative process and a classical Runge-Kutta method for Eq. 5.

We now turn to the evaluation of experimentally observable quantities. The easiest signal to measure is the total power emitted by the cloud. Neglecting radiation trapping effect at the frequency of the pump and repump lasers, energy conservation arguments allow to show that all the scattered light has to be deducted from the incident lasers beams, corresponding to the attenuation of the pump and repump lasers:

$$P_F = \int n(r) 4\pi r^2 dr [\sigma_{ext}^P(r) I_P + \sigma_{ext}^{Rep}(r) I_{Rep}], \quad (7)$$

where $\sigma_{ext}^P(r)$ (resp. $\sigma_{ext}^{Rep}(r)$) is the extinction cross section of the pump (resp. repump) laser. These cross sections can be obtained using the results of the optical

Bloch equations, as done for the extinction cross section of the light around the $F = 2 \rightarrow F' = 2$ line (see eq. 6). It is however possible to show that this emitted power P_F can also be written in a different way:

$$P_F = \int n(r) 4\pi r^2 dr [\hbar\omega_0 \Gamma(\rho_{2'2'}(r) + \rho_{3'3'}(r)) - \sigma_{ext}(r) I_{RT}(r)] \quad (8)$$

This expression highlights the emission due to the population of the excited state, corrected by the attenuation of the light which is subject to radiation trapping. We will exploit this later expression, as it also to compute not only the total emission, but also the relative spectral component along each optical transition. In figure 2 we plot the computed value of this total emitted power computed with the additional assumption of a cloud of constant density. Despite the approximations performed to arrive at these self-consistent equations, this model gives satisfactory results in agreement with our experimental observations.

Using this self-consistent model, we can compute additional observables, such as the population of different hyperfin ground states and the power emitted along each lines of the various optical transitions. An important feature in our explanation of the impact of radiation trapping on the emission of our cloud of atoms, is the increased population of the $F = 3$ hyperfin state. In contrast to astrophysical observations, we can measure the populations of the ground state of the atoms after swichting off all incident laser beams and that the (small) excited state population has relaxed to one of the two hyperfin ground states). The experimental result and the theoretical prediction are shown in figure 3, illustrating the very satisfactory agreement between the model and the experiments.

A further confirmation of our qualitative explanation of the impact of radiation trapping in the emission of an optical thick cloud of cold atoms can be obtained from the analysis of the spectrum of the emitted light. Using expression (8), we have computed the emission along each of the 4 possible optical transitions. All lines are directly proportional to the excited state population taking into account the respective branching ratios, expect the line along the $F = 2 \rightarrow F' = 2$ transition which needs to be corrected due to the radiation trapping effect. The experimental observation of the spectrum of the emitted light in cold atom experiments is technically more challenging, but with a moderate resolution we have been able to resolve the different lines of the relevant transitions. Figure 4 shows the experimental and the numerical results, in agreement with the qualitative explanation, that in our regime, the steady state population affected by radiation trapping leads to a larger population in the $F = 3$ ground state and corresponding increased fluorescence along the $F = 3 \rightarrow F' = 2$ line.

C. Beyond Rate Equations for the atomic response

Our simple model, using stationary rate equations coupled with one diffusion equation, agrees qualitatively with our experimental results. Despite this satisfactory result, several limitations of our models might account for the quantitative differences between this simplified model and the experimental observation and restrict the range of validity of the method described above.

As we have neglected the Zeeman substructure of all hyperfine levels, polarizations effects and Raman scattering among different Zeeman sublevels are not taken into account. As the incident lasers are polarized, we expect some degree of polarization to remain, at least for moderate radiation trapping. We expect a more refined model as used in astrophysics [28] will allow to improve the description of the light emitted by the cloud of cold atoms and measurements of the polarization along the various emission lines will allow to test the regime of validity of such more evolved codes for radiation trapping.

Another assumption which has simplified the model we have implemented related to the fact that we only consider radiation trapping along one single optical line. This allows for using a single diffusion equation self-consistently coupled to the atomic response. We do not expect a principle limitation to extend this approach to coupling to several diffusion equations. In the regime of parameters we have performed the experiments however, the use of a single diffusion equation is justified by the different optical thicknesses along the different atomic transitions. Neglecting saturation effects (which are most likely to occur on the close to resonance detuned strong pump transition), we can approximate the optical thickness along each line by

$$\begin{aligned} b^{32'} &\approx \frac{b_0 \rho_{33}}{1 + 4 \frac{\delta_P^2}{\Gamma^2}} = b_0 \rho_{33} \\ b^{22'} &\approx \frac{b_0 \rho_{22}}{1 + 4 \frac{\delta_P^2}{\Gamma^2}} = b_0 \rho_{22} \\ b^{23'} &\approx \frac{b_0 \rho_{22}}{1 + 4 \frac{\delta_{Rep}^2}{\Gamma^2}} = \frac{b_0}{65} \rho_{22} \\ b^{33'} &\approx \frac{b_0 \rho_{33}}{1 + 4 \frac{\delta_{Rep}^2}{\Gamma^2}} = \frac{b_0}{65} \rho_{33}, \end{aligned} \quad (9)$$

where $b^{ij'}$ is the optical thickness of the transition between $F = i$ and $F' = j$.

To justify that we use only one trapped light, we can compare optical thicknesses of the different lines. Four lines will be considered : $3 \rightarrow 2'$ (pump line), $2 \rightarrow 2'$ (the only one radiation trapping line considered), $2 \rightarrow 3'$ (repump line) and $3 \rightarrow 3'$ line. When $b_0 \gg 1$ ($L \gg l_{sc}$) diffusive behaviour of the light requires radiation trapping of this line to be taken into account. According to experiment results shown on figure 3, for the lower values of b_0 ($b_0 = 23$), we have $\rho_{33} \approx 4\%$ so we have corresponding line optical thicknesses : $b^{32'} \approx 1$, $b^{22'} \approx 23$, $b^{23'} \approx 0.35$

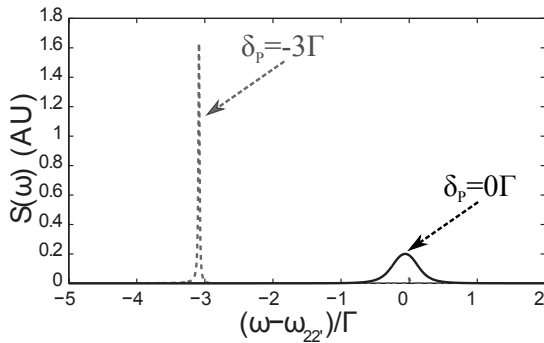


FIG. 5: Emission spectrum of an isolated atom close to the $2 \rightarrow 2'$ transition, computed from optical Bloch like equations as stated by the quantum regression theorem. The repumper has a Rabi frequency of 2Γ and a detuning $\delta_{Rep} = -5\Gamma$; the Rabi frequency of the pump is 2Γ and its detuning 0 (continuous line) and -3Γ (dotted line). While the atomic emission spectrum has a width comparable to the transition width when the pump is resonant, it becomes significantly narrower when it is detuned by a few Γ .

and $b^{33'} \approx 0.015$. In this regime, the dominant radiation trapping occurs on the $F = 2 \rightarrow F' = 2$ transition where we have used the diffusion equation. For larger values of b_0 however ($b_0 = 68$), $\rho_{33} \approx 10\%$ and $b^{32'} \approx 7$, $b^{22'} \approx 61$, $b^{23'} \approx 1$ and $b^{33'} \approx 0.1$. The optical thickness of the pump line ($F = 3 \rightarrow F' = 2$) needs the study of radiation trapping of this line, even though saturation and corresponding bleaching effects are expected to reduce the impact of radiation trapping. We note that such non linear effects are properly taken into account in the self-consistent set of equations as long as our rate equation model is valid.

Another assumption made in the approach we have used in this work concerns the coherence properties of diffuse light. In our model we have assumed rate equations to render correct account of the light-atom coupling. In particular this assumes that no atomic coherence between different (hyperfine or Zeeman) ground states are produced. Having neglecting the Zeeman structure of the atomic levels, we can investigate to what extend the light scattered along the $F = 2 \rightarrow F' = 2$ line could be able to induce optical coherences and thus induce coherences between the different hyperfin level in the ground state. Applying the quantum regression theorem to multilevel

atoms [36, 37], we have computed the optical spectrum of the light emitted along the $F = 2 \rightarrow F' = 2$ transition. As shown in figure 5 the spectrum of light emitted by an isolated atom driven close to resonance has a width of the order of the natural linewidth Γ . The increase of the total fluorescence as a function of the optical thickness is thus occurring in a regime where the linewidth of the scattered light is of the order of the width of the excited state, making the use of an incoherent radiation trapping model a reasonable approach. However when the pump laser is detuned further away from resonance, the emission line can become significantly narrower than the natural width of the transition. We therefore expect Raman coherences between the different hyperfin ground states to play a more prominent role when the pump laser will be detuned from the $F = 3 \rightarrow F' = 2$ transition, allowing even hyperfin Raman gain to appear. This situation is explored in our work on random lasing with cold atoms where gain and scattering needs to be combined.

IV. CONCLUSION

In this paper, we have demonstrated that radiation trapping in a cloud of cold atoms can significantly alter the emission properties of atomic clouds. Simple and strong evidence of radiation trapping can be thus obtained in a steady-state regime, in contrast to studies of radiation trapping exploiting trapping times [5]. Cold atom clouds thus appear as excellent candidates to investigate in a laboratory regimes of radiation trapping out of thermal equilibrium, characterized by a strong coupling between radiation and atomic populations. In the case considered in this work, a simple model coupling in a self-consistent way a diffusion equation (describing light transport) and rate equations (describing the atomic behavior) has allowed to explain qualitatively all our observations.

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